一种新的星载 GPS定轨滤波算法

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摘 要: 针对传统 EKF(Extended Kalman Filtering)算法应用于星载 GPS(Global Positioning System)低轨卫星定轨时系统噪声方差初值不易确定的问题,提出了一种新的定轨滤波算法. 该算法在非线性方程线性化过程中,在前一时刻滤波估值点进行线性化,从而得到扰动方程,并将该扰动方程引入到传统 EKF进行滤波处理. 该算法与传统 EKF分别应用在星载 GPS低轨卫星的定轨中,通过比较,结果表明改进的算法在一定程度上抑制了由于系统噪声方差阵选取偏差较大而引起的滤波发散现象,且对于系统噪声方差的初值选取有较强的鲁棒性.

关 键 词: 星载 GPS定轨; 扩展卡尔曼滤波; 滤波发散; 算法; 鲁棒性

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Novel filter algorithm of space-borne GPS orbit determination

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Abstract: When conventional EKF (extended Kalman filtering) algorithm was used in LEO (low earth orbit satellite) orbit determination by space-borne GPS (global positioning system). It is difficult to know the initial value of system noise variance. Therefore, the novel filter algorithm of orbit determination was proposed. In the new algorithm, the methods of linearization at the prior filtering estimate value was adopted during the linearization process of the nonlinear equations, and the disturbing equations were obtained, and then the disturbing equations were introduced in conventional EKF. Apply the EKF and the modified algorithm in LEO orbit determination based on space-borne GPS respectively, the results show the modified algorithm can restrain filter divergence of conventional EKF caused by the large deflection of system noise variance in a certain extent, and has better robustness to the selecting of initial value of system noise variance.

Key words: space-borne global positioning systrem; extended Kalman filtering; filtering divergence; algorithm; robustness

卡尔曼滤波因其能够根据得到的新的观测数据,随时计算新的参数滤波值,便于实时地处理观测数据,因此,被广泛地用于 GPS(Global Positiorning System)动态数据的处理中[1]. 而传统卡尔曼滤波仅适用于线性系统,对于星载 GPS,就只能采用 Bucy等人提出的非线性条件下的扩展的卡尔曼滤波 EKF (Extended Kalman Filtering)[2].

由于经典的 EKF对于动态系统状态噪声的鲁棒性很差,为此,本文提出了改进的 EKF算法,该算法把干扰偏差卡尔曼滤波方法中对偏差的估计应用于 EKF算法中,能在一定程度上抑制滤波发散现象. 经实验验证,结果表明改进的 EKF算法对于状态噪声方差初始值的选取具有较强的鲁棒性.

1 低轨卫星动态方程

1.1 状态方程

根据对低轨道卫星的摄动力分析,其主要影响为地球非球形的 J2项摄动,故本文采用低轨卫星 J2项影响的状态模型为

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$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -\frac{\mu x}{r^3}\left[1 - J_2\left(\frac{R_e}{r}\right)^2\left(7.5\frac{z^2}{r^2} - 1.5\right)\right] \\ -\frac{\mu y}{r^3}\left[1 - J_2\left(\frac{R_e}{r}\right)^2\left(7.5\frac{z^2}{r^2} - 1.5\right)\right] \\ -\frac{\mu z}{r^3}\left[1 - J_2\left(\frac{R_e}{r}\right)^2\left(7.5\frac{z^2}{r^2} - 4.5\right)\right] \\ \dot{b} \end{bmatrix} + \begin{bmatrix} W_x \\ W_y \\ W_z \\ W_{vx} \\ W_{vy} \\ W_{vz} \\ W_b \end{bmatrix} \tag{1}$$

式中，x, y, z为卫星在地心惯性坐标系中的三维位置坐标；$\dot{x}, \dot{y}, \dot{z}$为卫星在地心惯性坐标系中的速度分量；$\ddot{x}, \ddot{y}, \ddot{z}$为卫星在地心惯性坐标系中的加速度分量；$r$为地心距，$r = \sqrt{x^2 + y^2 + z^2}$；$\mu$为地球引力常数；$R_e$为地球赤道半径；$J_2 = 1.08263 \times 10^{-3}$；$b$为GPS接收机钟差；$\dot{b}$为接收机频差；$W$为各项对应的高斯白噪声.

对式(1)进行线性化求偏导后有如下矩阵：

$$\mathbf{A}_G(\mathrm{X}, t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ F_{11} & F_{12} & F_{13} & 0 & 0 & 0 & 0 \\ F_{21} & F_{22} & F_{23} & 0 & 0 & 0 & 0 \\ F_{31} & F_{32} & F_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

式中，F_{11}，F_{12}，F_{13}，F_{21}，F_{22}，F_{23}，F_{31}，F_{32}，F_{33}的取值参见文献[3]. 进行离散化后可得到如下的滤波状态方程：

$$\mathrm{X}_k = \phi_{k,k-1}\mathrm{X}_{k-1} + \mathrm{W}_k \tag{3}$$

式中，$\phi_{k,k-1}$为状态转移矩阵；W_k为状态噪声向量.

1.2 观测方程

GPS接收机伪距观测方程为

$$\rho_i = R_i + C \times b + \Delta\rho_i \quad (i = 1, 2, 3, \cdots, n) \tag{4}$$

式中，$R_i = \sqrt{(x_s^i - x_u)^2 + (y_s^i - y_u)^2 + (z_s^i - z_u)^2}$为观测到的第$i$颗GPS卫星到用户卫星间的几何距离；$C$为光速；$b$为接收机钟差；$\Delta\rho_i$为伪距误差.

在不考虑时钟频差的影响时，伪距率的观测方程为[4]

$$\dot{\rho} = \frac{(x_s^i - x_u)(\dot{x}_s^i - \dot{x}_u) + (y_s^i - y_u)(\dot{y}_s^i - \dot{x}_u)}{R_i} + \frac{(z_s^i - z_u)(\dot{z}_s^i - \dot{z}_u)}{R_i} + \Delta\dot{\rho}_i \quad i = 1, 2, \cdots, n \tag{5}$$

式中，(x_s^i, y_s^i, z_s^i)和$(\dot{x}_s^i, \dot{y}_s^i, \dot{z}_s^i)$为第$i$颗GPS卫星的位置和速度；$\Delta\dot{\rho}_i$为伪距率误差.

将式(4)、式(5)进行线性化并写成矩阵形式得到：

$$\mathrm{Z}(k) = \mathrm{H}(k)\mathrm{X}(k) + \mathrm{v}(k) \tag{6}$$

式中，$\mathrm{v}(k)$是量测噪声；$\mathrm{H}(k)$为观测矩阵；观测矩阵中的n为可见GPS卫星数.

$$\mathrm{H}(k)_{2n\times 7} = \begin{bmatrix} \frac{\partial R_1}{\partial x} & \frac{\partial R_1}{\partial y} & \frac{\partial R_1}{\partial z} & C & 0 & 0 & 0 \\ \frac{\partial \dot{\rho}_1}{\partial x} & \frac{\partial \dot{\rho}_1}{\partial y} & \frac{\partial \dot{\rho}_1}{\partial z} & \frac{\partial \dot{\rho}_1}{\partial \dot{x}} & \frac{\partial \dot{\rho}_1}{\partial \dot{y}} & \frac{\partial \dot{\rho}_1}{\partial \dot{z}} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial R_n}{\partial x} & \frac{\partial R_n}{\partial y} & \frac{\partial R_n}{\partial z} & C & 0 & 0 & 0 \\ \frac{\partial \dot{\rho}_n}{\partial x} & \frac{\partial \dot{\rho}_n}{\partial y} & \frac{\partial \dot{\rho}_n}{\partial z} & \frac{\partial \dot{\rho}_n}{\partial \dot{x}} & \frac{\partial \dot{\rho}_n}{\partial \dot{y}} & \frac{\partial \dot{\rho}_n}{\partial \dot{z}} & 0 \end{bmatrix} \tag{7}$$

2 改进的EKF算法

2.1 传统EKF算法

一般非线性离散系统的方程可由下式描述：

$$\left.\begin{aligned} \mathrm{X}_k &= f(\mathrm{X}_{k-1}, k-1) + \mathrm{W}_{k-1} \\ \mathrm{Z}_k &= h(\mathrm{X}_k, k) + \mathrm{V}_k \end{aligned}\right\} \tag{8}$$

式中，$f[\cdot]$是n维向量函数，对其自变量而言是非线性的；$h[\cdot]$是m维向量函数，它对自变量而言也是非线性的；W_k，V_k分别为n维和m维系统动态噪声和量测噪声. 对以上非线性系，可得到离散型非线性扩展卡尔曼滤波方程[5]：

$$\hat{\mathrm{X}}_{k/k-1} = \hat{\mathrm{X}}_{k-1} + \{f[\hat{\mathrm{X}}_{k-1}, t_{k-1}]\}T \tag{9}$$

$$\hat{\mathrm{X}}_k = \hat{\mathrm{X}}_{k/k-1} + \Delta\hat{\mathrm{X}}_k \tag{10}$$

$$\Delta\hat{\mathrm{X}}_k = K_k\{\mathrm{Z}_k - h[\hat{\mathrm{X}}_{k/k-1}, k]\} \tag{11}$$

$$\mathrm{K}_k = \mathrm{P}_{k/k-1}\mathrm{H}_k^{\mathrm{T}}[\mathrm{H}_k\mathrm{P}_{k/k-1}\mathrm{H}_k^{\mathrm{T}} + \mathrm{R}_k]^{-1} \tag{12}$$

$$\mathrm{P}_{k/k-1} = \phi_{k,k-1}\mathrm{P}_{k-1}\phi_{k,k-1}^{\mathrm{T}} + \mathrm{Q}_{k-1} \tag{13}$$

$$\mathrm{P}_k = (\mathrm{I} - \mathrm{K}_k\mathrm{H}_k)\mathrm{P}_{k/k-1}(\mathrm{I} - \mathrm{K}_k\mathrm{H}_k)^{\mathrm{T}} + \mathrm{K}_k\mathrm{R}_k\mathrm{K}_k^{\mathrm{T}} \tag{14}$$

初始条件为：

$$\left.\begin{aligned}\hat{X}_0 &= E[X_0] = m_{x_0}\\ P_0 &= Var(X_0) = C_{x_0}\end{aligned}\right\} \tag{15}$$

以上算法,应用于低轨卫星定轨时,在初始状态方差阵 Q选取合适时,滤波效果较好,但是,如果初值选取偏差大时,会出现滤波发散现象,滤波结果与真值相差很大. 为了克服以上不足,本文提出了改进的扩展卡尔曼滤波算法.

2.2 改进的扩展卡尔曼滤波

扩展卡尔曼滤波进行线性化时,采用围绕目前状态估值线性化,其线性化点与观测对象的运动状态相关,观测对象的动态噪声会影响到线性化的精度. 本文采用在上一步估计值的基础上进行线性化,此方法可以不受动态观测噪声的影响,便于数据处理,也能保证一定的精度. 然后将得到的干扰方程与扩展卡尔曼滤波算法相结合,便得到改进的扩展卡尔曼滤波算法. 下面就进行理论推导并给出改进的滤波方程.

定义[6]:

$$\Delta X_{k+1} = X_{k+1} - \hat{X}_k \tag{16}$$

式中, $\hat{X}_k$ 为目前时刻的估计值.

由式(8)中的状态方程,可得:

$X_{k+1} = f(X_k, k) + W_k$,将其在上一时刻状态最优估计点按泰勒级数展开,并取一阶项得:

$$X_{k+1} = f(\hat{X}_{k-1}, k-1) + \left.\frac{\partial f}{\partial X}\right|_{X=\hat{X}_{k-1}} \cdot (X_k - \hat{X}_{k-1}) + W_k$$

可表示为

$$X_{k+1} = \hat{X}_k + \Phi\Delta X_k + W_k \tag{17}$$

由式(17)可得到如下表达式:

$$\Delta X_{k+1} = \Phi\Delta X_k + W_k \tag{18}$$

同理对观测方程应用同样的方法得到:

$$\Delta Z_{k+1} = H\Delta X_{k+1} + V_{k+1} \tag{19}$$

结合式(18)、式(19),即可得到按上一步估计值线性化的干扰方程:

$$\left.\begin{aligned}\Delta X_{k+1} &= \Phi\Delta X_k + W_k\\ \Delta Z_{k+1} &= H\Delta X_{k+1} + V_{k+1}\end{aligned}\right\} \tag{20}$$

根据式(20)并结合扩展卡尔曼滤波算法,便得出改进的扩展卡尔曼滤波(MEKF)方程:

$$\Delta\hat{X}_{k,k-1} = \Phi_{k,k-1}\Delta\hat{X}_{k-1} \tag{21}$$

$$\hat{X}_{k,k-1} = \hat{X}_{k-1} + \{f[\hat{X}_{k-1}, t_{k-1}]\}T \tag{22}$$

$$P_{k,k-1} = \Phi_{k,k-1}P_{k-1}\Phi^T_{k,k-1} + Q_{k-1} \tag{23}$$

$$K_k = P_{k,k-1}H_k^T[H_kP_{k,k-1}H_k^T + R_k]^{-1} \tag{24}$$

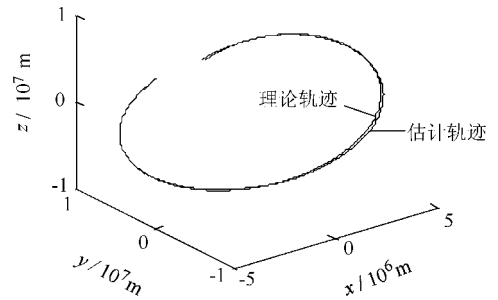
$$\Delta Z_k = Z_k - h[\hat{X}_{k,k-1}, k] \tag{25}$$

$$\Delta\hat{X}_k = \Delta\hat{X}_{k-1} + K_k(\Delta Z_k - H_k\Delta\hat{X}_{k-1}) \tag{26}$$

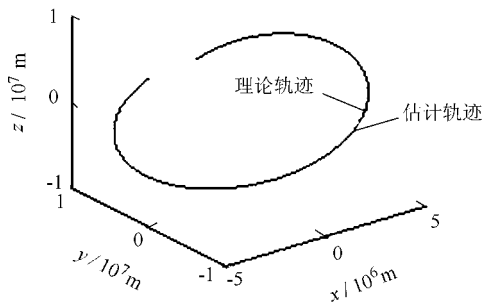
$$\hat{X}_k = \hat{X}_{k,k-1} + \Delta\hat{X}_k \tag{27}$$

3 计算分析

低轨卫星运动采用的模型为 J2摄动模型,其星历数据由美国 AGI公司开发的 STK(Satellite Tool Kit)卫星仿真工具包产生,仿真时间 6 000 s,观测噪声协方差阵 $R = diag(50^2, 0.2^2)$,用户卫星状态初值由最小二乘法先进行一次定位获得. 误差方差阵 P的初值取 $P = diag([500^2, 500^2, 500^2, 2^2, 2^2, 2^2, 10^{-6}])$,系统噪声方差阵 $Q = diag(10, 10, 10, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-6})$,在上述相同条件下,传统扩展卡尔曼滤波定轨与改进算法定轨结果分别如图 1~图 3所示.



a　EKF轨迹对照



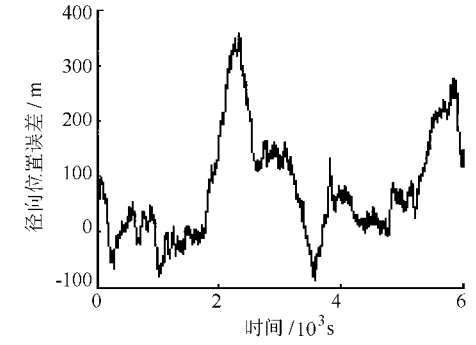
b　MEKF轨迹对照

图 1　两种滤波算法定轨轨迹对照图

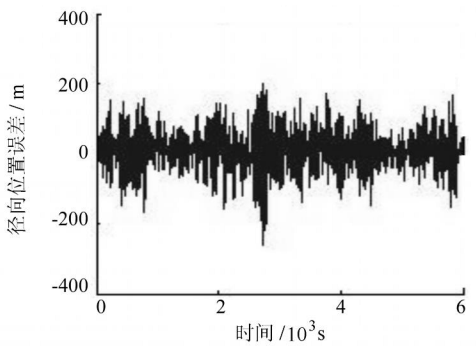
由图 1a、图 1b可以看出,EKF的理论轨迹与滤波估计轨迹在经过一段时间后产生了明显的偏差,而 MEKF算法的轨迹跟踪效果很好.

由图 2a、图 2b可以看出,此时 EKF算法的径向位置误差出现发散,该输出序列不再是一平稳序列;而 MEKF算法的径向位置误差还是平稳的,没有出现发散现象,误差均方差为 52.27 m.

由图 3a、图 3b看出,此时的 EKF算法的径向速度误差发散,而 MEKF算法的径向速度误差还是平稳的,没有出现发散现象,误差均方差为 0.11 m/s

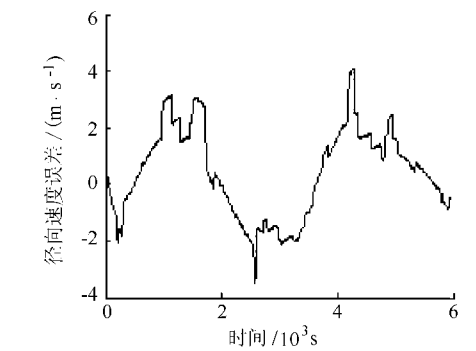


a EKF径向位置误差

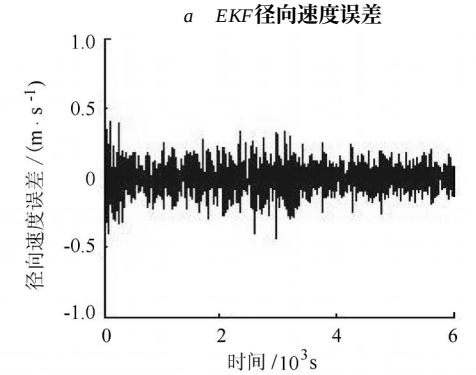


b MEKF径向位置误差

图 2 径向位置误差对照图



a EKF径向速度误差



b MEKF径向速度误差

图 3 径向速度误差对照图

4 结 论

本文针对星载 GPS数据的状态协方差阵 Q不好准确确定的问题,提出了在前一时刻滤波估值点进行线性化,并将利用此方法得到的扰动方程引入到扩展卡尔曼滤波算法中,得到改进扩展卡尔曼滤波算法,所提出的新算法对状态协方差阵 Q的鲁棒性较好,能在一定范围内抑制由于 Q阵选取不当带来的滤波发散问题,在系统动态噪声不确定情况下,本文提出的算法具有较强的适用性.

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